On the Deformation and Fracture of Granite under High Confining Pressure

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By

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Abstract

The stress-strain relations for granite under various high confining pressures were observed experimentally. Mean Young's modulus is numerically constant independent of increasing pressure. The volume increase in the fracture range, observed characteristically at an atmospheric pressure, decays with pressure. The empirical formula of pressure-strength relationship is given by

$$P^* \fallingdotseq P_0^*(kP_H+1)^{1/2}.$$

It seems that the phenomena above mentioned have the close connection with compressibility (i.e. porosity). The pressure-strength relation was calculated with use of Griffith's crack (pore) theory, putting the reasonable (or convenient) assumptions into calculation. This calculated relation, deduced from the empirical equation of compressibility with pressure, gives the same formula as above mentioned empirical one.

1. In recent years, a large number of experimental results on the deformation and fracture of rocks have been reported, but most of them were carried out for carbonate rocks such as marble and limestone (Griggs, et al., 1951, Turner, et al., 1954, Robertson, 1955, Paterson, 1958). These rocks exhibit the rheological properties the same as igneous rocks at an atmospheric pressure and room temperature. Under high pressure, however, these rocks flow plastically showing the distinct yielding zone, while the igneous rocks have scarcely the sign of plastic deformation up to the considerablly high pressure, at room temperature. The mechanism of the deformation and fracture of igneous rocks is much complicated as is observed about brittle substances in general. As the igneous rocks, however, have the close connection with the rheological behavior in the deeper part of the crust, it must be made throughly clear how these rocks deform and fracture under high pressure.

In the previous paper, the experimental results of the deformation and fracture for granite under an atmospheric pressure have been reported (Matsushima, 1960). Here, we report the experimental results under high confining pressure, then express a brief consideration upon the mechanism of the fracture phenomena.

Of course, in order to obtain the definite knowledges on the earth's interior, we must take into consideration the physical and the chemical states of the earth's interior such as temperature, its gradient and time-fluctuation, heat-flow and -generation, states of the internal stresses and their dependence on time, and the constituent substances. But we shall neglect all these effects and confine ourselves into the studies of deformation and fracture of granite.

2. The apparatus used for this experiment is the conventional triaxial testing cylinder as shown in Fig. 1. The shape and the size of specimen give the considerable affection upon the aspect of deformation and the strenghth. Especially as to the crystalline aggregate constructed with a large size of grains, such as granite, these effects may be strikingly large. Then the capacity of pressure vessel was taken as large as possible to be able to contain the large size of specimen. Therefore, the durability against pressure of this vessel was sacrificed inevitably. This vessel can endure up to 5,000 atm., and axial stress can be produced by the 300 ton press. All the specimens were enclosed to prevent the confining liquid penetrating into the specimens. Synthetic adhesive rubber was used as covering material, for its good insulating character and flexibility. The strain was measured by the strain gauges of electric resistance type, same as in the previous

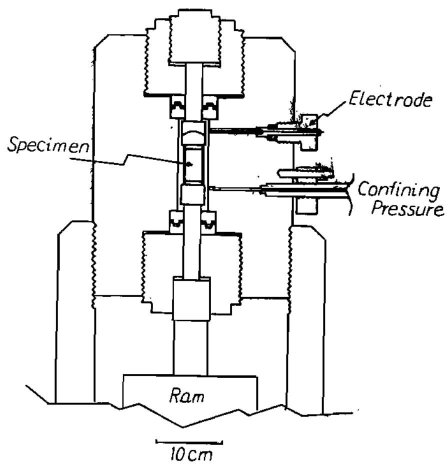

Fig. 1. Triaxial Testing Cylinder.

experiments. Accuracy of the measurement was within 50 atm. as to the confining pressure, 100 kg/cm² as to the axial compressional stress and 10^{-5} as to the strain.

3. The relations between the axial compressional stress and the strain under various confining pressures are shown in Fig. 2. The strains were measured in the axial direction and along the circular arc of the cylindrical specimen. The long columns of Kitashirakawa biotite granite, 30 mm. in diameter and 60 mm. long were used up to 1800 atm. confining pressure, 25 mm. in diameter 50 mm. long up to 3800 atm., and 20 mm. in diameter 50 mm. long above 3800 atm.. The slender specimen has a tendency to bend, and the stumpy one may deform into barrel shape. In these experiments, it was scarecely observed that the specimens deform in barrel shape within the elastic range. Near the rupture point, however, the plastic flow was observed, though it was not considerable, and a slight barrel shape deformation was recognized.

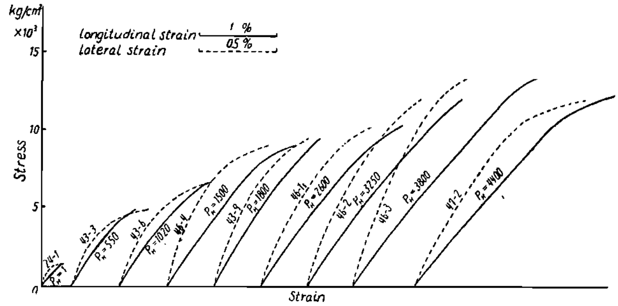

Fig. 2. Stress-Strain Curves from Axial Compression Tests for Kitashirakawa Granite.

Each full line denotes the longitudinal direction and broken line the lateral direction.

The stress-strain curves in the longitudinal direction have the approximately constant slope, independently of confining pressure, as shown in Fig. 2. That is, it can be said that the mean Young's modulus of this rock is constant at any confining pressure. In the fracture range under enough high pressure, however, as above mentioned, the curves have the tendency to be concave toward the strain axis and show the indistinct yielding zone, producing the appreciable plastic flow.

In Fg. 3, the relations between the stress and preudo-Poisson's ratio under various confining pressures are shown. Where percentage stress is used as abscissa, the percentage stress is defined as a hundred times of the ratio of stress to the rupture strength. The stress and Poisson's ratio relations under various pressures can be clearly shown by the use of percentage stress. With the increase of the confining pressur, the unusual lowness of pseudo-Poisson's ratio at the first stage of loading is gradually lost and the volume increasing effect in the fracture range, observed characteristically at the ordinally pressure (Bridgman, 1949), decays rapidly.

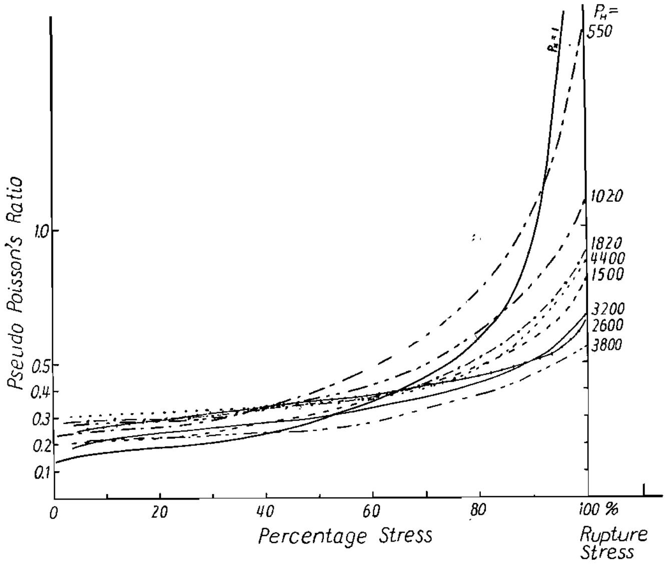

Fig. 3. Pseudo-Poisson's Ratio (ratio of lateral strain to longitudinal strain) vs. Percentage Stress under Various Confining Pressures.

Rupture strength (approximately equal to yield strength) increases strikingly with the increase of pressure. The increment of strength, however, was not so considerable at fairly high pressure. The observed values of strength at various pressures are shown in Fig. 4(a). Blanc circles express the values for the specimens in the ratio of length to diameter, 2 : 1, and full circle, 2.5 : 1.

The empirical formula which expresses the pressure-strength relation is,

$$P^* = P_0^*(kP_H + 1)^{1/2},$$

where P^* is the strength, k is the constant and P_H is the confining pressure.

The important numerical results are listed in the following Table.

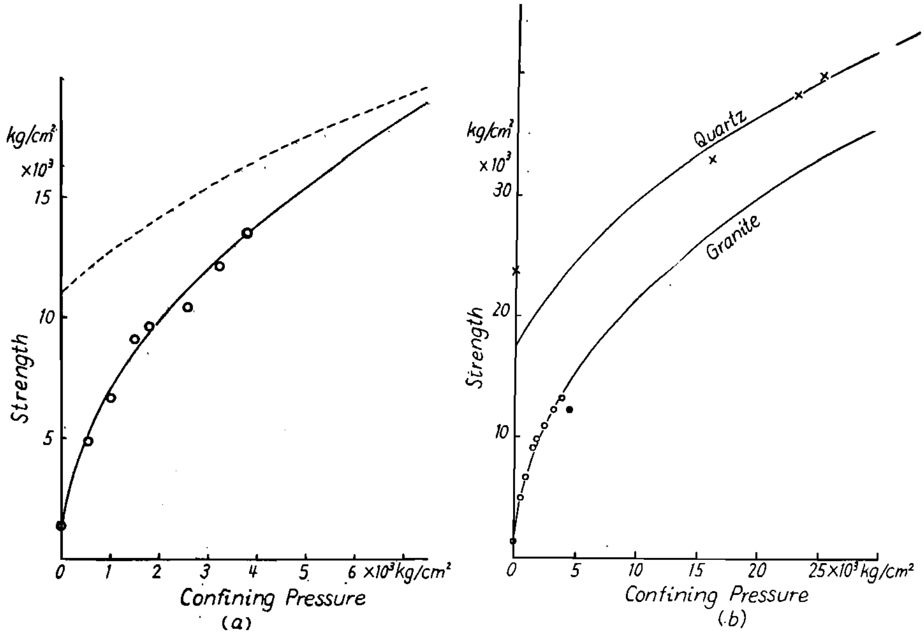

Fig. 4. Strength vs. Confining Pressure.

(a) Observed values and empirical curve (full line) of strength for Kitashirakawa granite, and assumed strength (broken line) as the pores are perfectly closed.

(b) Extended figure of (a), and strength of quartz.

Table. Mean Young's Modulus, Rupture Strength, and Indistinct Yielding Point.

Confining Pressure atm.	Mean Young's Modulus		Rupture Strength kg/cm²	Yielding Point (indistinct) kg/cm²
	$E\times10^{-11}$ dyne/cm²	Stress Range kg/cm²		
1	5.4	0– 1250	1380	—
550	6.9	3960	4860	4200
1020	6.25	5940	6660	6000
1500	6.7	– 7800	9100	8300
1800	7.5	– 8530	9160	—
2600	6.5	– 8580	10400	8600
3250	6.25	–11600	12110	—
3800	6.2	–12500	13520	12500
4400	6.0	–10100	12150	10900

On the other hand, the strength of quartz measured by Bridgman (1952) is shown by mark X in Fig. 4(b). The strength of this substance is relatively little affected by the pressure and only slightly increases as pressure does. The pressure-strength relation of granite at considerablly high pressure seems to become the same as that of quartz by the reason after Griffith's theory (1924). Therefore, the assumed P^*-P_H curve can be obtained by extrapolating the above relation toward the lower pressure. This curve is shown by the broken line in Fig. 4(a).

We regard the difference between the assumed curve and the observed value as "the strength lowering" $P^{*\prime}$, considering that the strength is lowered by a certain cause characteristic of granite.

Now, we compare the change of "the strength lowering" and the volume increase in the fracture stage with the decrease of compressibility (Adams, 1951). As shown in Fig. 5, there is a close connection among these phenomena. As the compressibility decrement with the increase of pressure may be caused by the closing of pores, it is reasonably considered that the strength may be strongly affected by the existence of pores. In Fig. 5, the volume increment S in the fracture range show the area of the part where the pseudo-Poisson's ratio goes over 0.5 in Fig. 3 as the index.

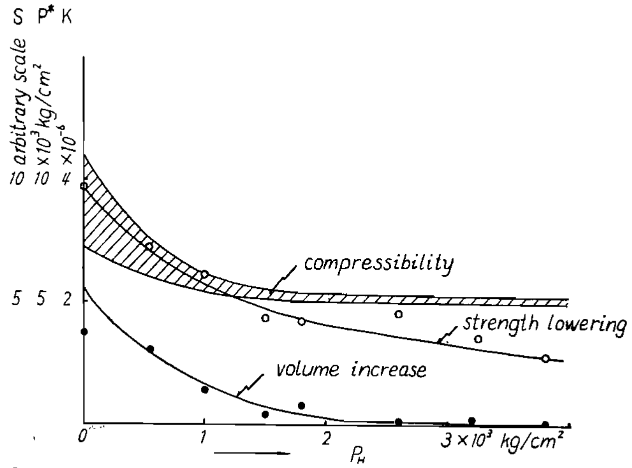

Fig. 5. Comparison of "Strength Lowering", Volume Change in the Fracture Range and Compressibility, for Granite under Various Pressures.

4. We will introduce one representation with respect to the relationship between the strength and the prosity of granite under various confin-

ing pressures, it will be considered that these two have a close connection with each other.

We assume that the pressure is able to affect to the porosity alone, but not to its strength. Then the rupture can be directly produced only by the differential stress acting on the porous specimen. Of course, the pressure exerts the indirect effect upon the strength, changing the shape of the pores.

Compressibility κ is given as follows,

$$\kappa=-\frac{1}{V}\frac{dV_1}{dP_H}-\frac{1}{V}\frac{dV_2}{dP_H}=\kappa_1+\kappa_{por},\quad V_1\gg V_2,\ \cdots\cdots\cdots\cdots(1)$$

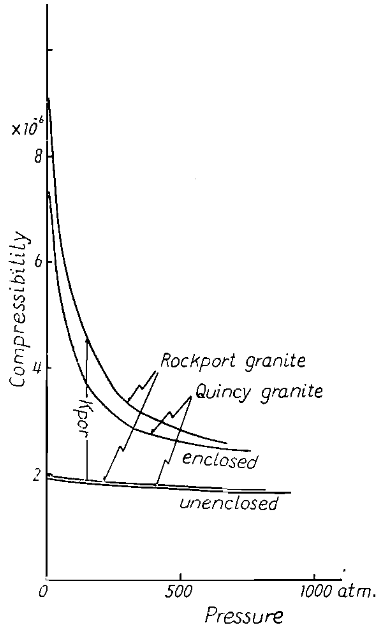

Fig. 6. Compressibility of Enclosed and Unenclosed Specimens of Granite under Various Pressures (Zisman, 1933).

where V, V_1, V_2, the total volume of specimen and the volume of substantial and empty part respectively, κ_1 denotes the compressibility of material and κ_{por} the effective value with respect to the pores.

Put N the number of the pores contained in this specimen, and v_1, v_2, $\cdots\cdots v_N$ the volumes of the respective pores, then

$$\kappa_{por}=-\frac{1}{V}\frac{d}{dP_H}\Big(\sum_{i=1}^{N}v_i\Big).\ \cdots\cdots\cdots(2)$$

The empirical formula of κ_{por} is given as follows

$$\kappa_{por}=\frac{1}{(mP_H+n)^2}=\frac{1}{n^2(kP_H+1)^2},$$

$$k=\frac{m}{n},\qquad\cdots\cdots\cdots\cdots\cdots(3)$$

from the experiment on the measurements of the compressibility by Zisman (1933), considering the difference between the compressibility of the covered specimen and the uncovered one.

Now, we assume that the contraction of the respective pores with pressure has the similar tendency, then the effective compressibility of i-th pore in this specimen is given by

$$\left.\begin{aligned}\kappa^{t}_{por}&=\frac{1}{(m_iP_H+n_i)^2}=\frac{1}{n_i^2(kP_H+1)^2}\,,\\ k_i&=\frac{m_i}{n_i}=k,\quad \sum_{i=1}^{N}\frac{1}{n_i^2}=\frac{1}{n^2}\,.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

This formula (4) corresponds to the i-th term of formula (2), then

$$-\frac{1}{V}\frac{dv_i}{dP_H}=\frac{1}{n_i^2}\frac{1}{(kP_H+1)^2}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(5)$$

Integrating and put the condition $P_H\to\infty$, $v_i\to 0$ into account,

$$v_i=\frac{V}{m_in_i}\cdot\frac{1}{(kP_H+1)}\,.\qquad\cdots\cdots\cdots\cdots\cdots\cdots(6)$$

On the other hand, we assume the shape of pore as ellipsoid rotated around the minor axis, then the volume is

$$v_i=\frac{4\pi c_i^2b_i}{3}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

where c_i and b_i are the major and minor radius respectively.

Equating formula (6) and (7), and assumeing that c_i does not vary with pressure, then the relation of the length of b_i with pressure is given by

$$b_i=\frac{3}{4\pi c_i^2}\cdot\frac{V}{m_in_i}\cdot\frac{1}{(kP_H+1)}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

Next, we try to obtain the critical stress p_i, according to Griffith's theory (1921), at which the elliptic pore or crack begin to elongate by the applied compressional stress P along major axis, as is shown in Fig. 7. We take elliptic co-ordinates α, β and put $\alpha=\alpha_{0i}$ at the boundary of the i-th crack. The major and minor radius is given by c_i and $b_i=\alpha_{0i}c_i$, putting c_i the half length of focal line, as α_{0i} is very small.

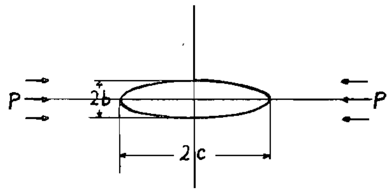

Fig. 7. Elliptic Pore Exposed under Axial Compressional Stress P along the Major Axis.

Then the increase of strain energy W_i by such crack is given by

$$W_i=\alpha_{0i}\frac{\pi P^2c_i^2}{4E}(1-\sigma),\qquad\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

where E, σ are the Young's modulus and Poisson's ratio of this material respectively. On the other hand, the surface energy of this crack is

$$U_i=4c_iT,\qquad\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

where T is the surface energy per unit area. From the condition that the crack may extend

$$\frac{d}{dc_1}(W_t - U_t) = 0, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(11)$$

the strength of this crack can be expressed as

$$p_t^* = a\left(\frac{ET}{\pi(1-\sigma)}\right)^{1/2} \cdot b^{-1/2}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(12)$$

where a is the numerical constant (Ono, 1949).

Substituting (8) into (12), the relation of confining pressure and the strength of the crack oriented to the direction of compressional stress is given by

$$p_t^* = \frac{2}{\sqrt{3}} ac_t\left(\frac{ET}{(1-\sigma)} \cdot \frac{m_t n_t}{V}\right)^{1/2} (kP_H + 1)^{1/2}$$
$$= p_{t0}^*(kP_H + 1)^{1/2}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(13)$$

where p_{t0}^* is the strength of this crack at $P_H = 0$.

The value of k obtained from the above stated empirical equation is

$$k = 2.49\ 10^{-2} \times (\text{kg/cm}^2)^{-1}.$$

Though the value of k for Kitashirakawa granite from the compressibility measurements has not been given, the values of k from Zisman's data are

$$k = 0.22\ 10^{-2} \times (\text{kg/cm}^2)^{-1}$$

for Quincy granite, and

$$k = 0.30\ 10^{-2} \times (\text{kg/cm}^2)^{-1}$$

for Rockport granite.

5. The mechanism of deformation and fracture of granite becomes much simpler at high confining pressure, as seen in the case of quartz and so on, than at lower pressure, by the action of hydrostatic compression which decreases the porosity of porous media, while such rocks show a very complicated behavior under low pressure. This tendency may be kept at high temperature and high pressure. Therefore, in the deeper part of the crust where rocks are confined with enough high pressure, the feature of mechanical behavior of rocks seems to have lost their complicacy such as seen at the earth's surface, and may behave as a simple aggregate of constituent minerals.

So far as it is concerned at room temperature, these sorts of rocks are not so much plastic even under high pressure. On the contrary, the elastic range is extended strikingly with the increase of pressure as the results of the raise of strength. However, if the pressure increases highly enough, the increment of strength does not become so remarkable, then the elevation of temperature may become more strongly effective on the plasticity than the increase of pressure, in the earth's interior.

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